Mathematische Methoden zur Analyse von Zeitreihen komplexer Systeme

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Aufgabenblatt

Aufgabe 1 Observational noise and parameter estimation

- Generate realizations of length $N = 1000$ of an AR[1] process:

$$x(t) = ax(t-1) + \epsilon(t), \quad \epsilon(t) \sim N(0,1),$$

  with $a = 0.98$, corresponding to a relaxation time of approximately 50 time steps.

- Estimate the parameter a by

$$\hat{a} = \frac{\sum_t x(t-1)\, x(t)}{\sum_t x(t)\, x(t)} \qquad (1)$$

- Add Gaussian white observational noise of standard deviation 1, 5 and 10 to the realized time series and estimate the parameter a based on Eq. (1).

- Estimate the parameter a based on the state space model

$$\begin{aligned} x(t) &= a\, x(t-1) + \epsilon(t), \\ y(t) &= x(t) + \eta(t), \end{aligned}$$

  by the EM-algorithmus for the above three types of time series.

- Understand the result in terms of the signal-to-noise ratio.