Aufgabenblatt

Aufgabe 1 Observational noise and parameter estimation

• Generate realizations of length $N = 1000$ of an AR[1] process:
  \[ x(t) = ax(t-1) + \epsilon(t), \quad \epsilon(t) \sim N(0, 1), \]
  with $a = 0.98$, corresponding to a relaxation time of approximately 50 time steps.

• Estimate the parameter $a$ by
  \[ \hat{a} = \frac{\sum_t x(t-1)x(t)}{\sum_t x(t)x(t)} \quad (1) \]

• Add Gaussian white observational noise of standard deviation 1, 5 and 10 to the realized time series and estimate the parameter $a$ based on Eq. (1).

• Estimate the parameter $a$ based on the state space model
  \[
  \begin{align*}
  x(t) &= ax(t-1) + \epsilon(t), \\
  y(t) &= x(t) + \eta(t),
  \end{align*}
  \]
  by the EM-algorithmus for the above three types of time series.

• Understand the result in terms of the signal-to-noise ratio.