Statistics and Numerics

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Exercise Sheet Nr. 7

Exercise 1: Maximum entropy method

With the maximum entropy method (MEM), it is possible to construct distributions that comply with specific constraints, and contain the least a priori information. The entropy is given by

$$S = -\sum_{i=1}^{N} p_i \log p_i \ . \tag{1}$$

a) Maximize Eq. (1) analytically with respect to the probabilities p_i of N discrete entities $1, 2, \ldots, i, \ldots, N$. Include the prior knowledge,

$$\sum_{i=1}^{N} p_i = 1 \ , \tag{2}$$

by constrained optimization (Lagrange multiplier).

- Given the symmetry of Eq. (1), what can you state about the solution?

b) Choose $N = 10$ and use Powell's method for a numerical optimization of Eq. (1). Note that this algorithm is normally used for minimization. Implement Powell's method for entropy maximization (Eq. (1), subject to Eq. (2)):

- Implement a function `linmin(f,a,c,tol)` that performs a golden-section search for a 1-dimensional function $f(x) \in \mathbb{R}$, $x \in [a, c] \subset \mathbb{R}$. Use the tolerance `tol` to terminate the search algorithm once the search interval length falls below the threshold.
- Test `linmin` with the function $f(x) = x^2$. Does it work and what happens if the minimum of $f(x)$ is not within $[a, c]$?
- Create a two-step parameter transformation $\vec{p} = \Theta(\vec{x})$, $\vec{x} \in \mathbb{R}^N$, satisfying the constraints $0 \leq p_i \leq 1$ and $\sum_{i=1}^{N} p_i = 1$. The first step transforms $x_i \in \mathbb{R} \to [0, 1]$, and the second normalizes $\vec{p}$.
- Use `linmin` and the parameter transformation to minimize the function

$$-S(\vec{x}) = \sum_{i=1}^{N} p_i \log p_i = \sum_{i=1}^{N} \Theta_i(\vec{x}) \log \Theta_i(\vec{x})$$

with respect to $\vec{x}$. Copy the code of the solution from the lecture homepage (line 65ff.) and try to follow the procedure of Powell's method.