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Measuring the solar meridional flow from perturbations of eigenfunctions of global oscillations

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We present a new concept to infer the meridional flow in the deep solar interior from global oscillation data. This concept is based on the estimation of the coupling strengths of p-mode eigenfunctions mediated by the meridional flow. We illustrate the performance of the method using simulations and present first inversion results of a large-scale flow component of the meridional flow obtained from MDI data.

1 Introduction

The meridional flow is a large-scale poloidal flow in the Sun that is associated to the solar cycle activity. It is assumed that the flow extends from the surface down to the bottom of the convection zone. Location and strength of the return flow are of importance for the understanding of the solar dynamo (Küker, Rüdiger & Schultz 2001; Nandy & Choudhuri 2002). If one aims to infer the meridional flow by helioseismology in deeper layers one must analyze solar acoustic modes of small harmonic degree l which penetrate deep into the Sun. In Schad, Timmer & Roth (2011) we proposed a theoretical framework for the inversion of the meridional flow using the perturbation of mode eigenfunctions of global oscillations of low and medium degree. Here we present the application of this framework for the inversion of the meridional flow. We illustrate the performance of the method using simulated data and present first inversion results of a large-scale component of the meridional flow obtained from MDI data covering the upper part of the convection zone.

2 Theory

The application of quasi-degenerate perturbation theory to the solar oscillation equations shows that the presence of a meridional flow $\boldsymbol{u}$ results in a perturbation of eigenfunctions $\boldsymbol{\xi}_k^0$ and eigenfrequencies ω_k of p-modes of a flow free solar model.

2.1 Coupling of mode eigenfunctions

For each target mode $k = (n, l)$ that is specified by its radial order n and harmonic degree l one can find a set of coupling modes $k' = (n', l') \in K_k$ such that the contribution of the perturbed mode k to the global oscillation velocity $\boldsymbol{v}$ field at time t and position (r, θ, ϕ) is given by

$$\boldsymbol{v}_k(r,\theta,\phi,t) \;=\; \alpha_k(t) \sum_{k' \in K_k} c_{kk'} \boldsymbol{\xi}^0_{k'}(r,\theta,\phi)\,, \tag{1}$$

where $\alpha_k(t)$ is the oscillation amplitude and $\boldsymbol{\xi}^0_{k'}$ the unperturbed eigenfunction of mode k' as obtained from a flow free solar model. The expansion coefficients $\{c_{kk'}\}$ represent the coupling coefficients which can be approximated up to first order by

$$c_{kk'} \;\approx\; \begin{cases} 1 & \text{for} \quad\; k' = k \\ \frac{H_{k'k}}{\omega_k^2 - \omega_{k'}^2} & \text{for} \;\; k' \in K_k \backslash \{k\} \end{cases}, \tag{2}$$

where $H_{k'k}$ denotes the general matrix element

$$H_{k'k} = -2\,\mathrm{i}\,\omega_{\mathrm{ref}} \int \rho_0\, \overline{\boldsymbol{\xi}^0_{k'}} \cdot (\boldsymbol{u} \cdot \nabla \boldsymbol{\xi}^0_k)\, \mathrm{d}^3\boldsymbol{r} \in \mathbb{C}\,. \tag{3}$$

The matrix element represents the coupling of mode k' and k mediated by the flow. Due to the properties of the flow it is a purely imaginary number where its magnitude refers to the coupling strength. Analogously one finds that the mode frequency ω_k is shifted due to the meridional flow by

$$\delta\omega_k \approx \frac{1}{2\,\omega_{\mathrm{ref}}} \sum_{k' \in K_k \backslash \{k\}} \frac{|H_{k'\,k}|^2}{\omega_k^2 - \omega_{k'}^2}\,, \tag{4}$$

which is of second order in the matrix element $H_{k'k}$.

For our investigations it is convenient to represent the meridional flow in terms of

$$\boldsymbol{u}(\boldsymbol{r}) = \sum_{s=1}^{\infty} \left[u_s(r) Y_s^0(\theta,\phi) \boldsymbol{e}_r + v_s(r) \partial_\theta Y_s^0(\theta,\phi) \boldsymbol{e}_\theta \right], \tag{5}$$

where Y_s^0 is the spherical harmonic function of degree s and order $t = 0$. The degree s specifies the number of flow cells over latitude, e.g. the $s = 2$ component consists of

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two flow cells, one on each hemisphere. In this representation the flow is decomposed into a radial component of flow strength $u_s(r)$ and a horizontal component of flow strength $v_s(r)$. The flow strengths of each degree s are related with each other due to the conservation of mass as (Lavely & Ritzwoller 2008)

$$\rho_0 r\, s(s+1) v_s = \partial_r (r^2 \rho_0 u_s)\,, \tag{6}$$

where ρ_0 is the mass density of the unperturbed solar model. Hence, knowing the radial flow strength one can also derive the horizontal flow strength.

Inserting Eqs. (5) and (6) into Eq. (3) it is possible to expand the general matrix element by an orthogonal set of polynomials $\mathcal{P}^s_{l'l}(m)$ over azimuthal order m as

$$H_{k'k} = H_{n'l',nl}(m) = \mathrm{i}\,\omega_{nl} \sum_s b^s_{n'l',nl} \mathcal{P}^s_{l'l}(m)\,. \tag{7}$$

The polynomials refer to the coupling of angular momentum and are defined as

$$\mathcal{P}^s_{l'l}(m) := (-1)^{-m} \begin{pmatrix} l' & s & l \\ -m & 0 & m \end{pmatrix}\,, \tag{8}$$

where the term in brackets on the RHS denotes the Wigner-3j symbol (Edmonds 1974). The expansion coefficients define the b-coefficients which are related to the radial flow strength u_s by a linear integral equation.

$$b^s_{n'l',nl} = \int_0^R \rho_0(r) K_s^{n'l',nl}(r) u_s(r) r^2 \mathrm{d}r \tag{9}$$

with poloidal flow kernels $K_s^{n'l',nl}(r)$ given in (Schad et al. 2011). Consequently, knowing the b-coefficients one can infer the radial and the horizontal flow strengths of the meridional flow by means of Eqs. (9) and (6).

2.2 Observable effect on global oscillations time series

Global oscillations can be investigated from measurements of Dopplergrams $v_\mathrm{D} = \boldsymbol{v} \cdot \boldsymbol{e}$ in the solar photosphere where $\boldsymbol{v}$ is the solar velocity field and $\boldsymbol{e}$ is the line-of-sight axis of the observing instrument. Time series of the global oscillation amplitude of modes can be extracted from the Dopplergrams v_D by a spherical harmonic transformation. For this purpose the Dopplergrams are projected onto the spherical harmonic of selected degree l and azimuthal order m weighted by the instruments apodization function W

$$o_{l'm'}(t) \; = \; \int \overline{Y^{m'}_{l'}}(\theta,\phi) W(\theta,\phi) v_\mathrm{D}(\theta,\phi,t)\, \mathrm{d}\Omega\,. \tag{10}$$

The time series are related to the coupling coefficients $\{c_{kk''}\}$ as

$$o_{l'm'}(t) = \sum_k \alpha_k(t) \sum_{k''\in K_k} c_{kk''} L_{k'k''} U_{k''}(R)\,, \tag{11}$$

where $\{L_{k'k''}\}$ are the leakage matrix elements and $U_{k''}(R)$ is the radial eigenfunction of mode k'' at radius R where the absorption lines are formed. The leakage matrix results from observational restrictions of the velocity field which are essentially determined by the apodization function W and the projection of the solar velocity field onto the instruments line-of-sight axis $\boldsymbol{e}$ and other properties of the instrument. It origins from the fact that spherical harmonic functions are not orthogonal when not integrated over the complete solar surface. As a new observable we have introduced the amplitude ratio $y_{lm\,l'm}$ (Schad et al. 2011),

$$y_{lm\,l'm}(\omega_{nlm}) \; := \; \frac{\tilde{o}_{l'm}(\omega_{nlm})}{\tilde{o}_{lm}(\omega_{nlm})}\,, \tag{12}$$

which defines the ratio between the global oscillation amplitudes in the frequency domain $\tilde{o}_{lm}$ of two coupling modes evaluated at the oscillation frequency of the target mode. The amplitude ratio is related to the coupling coefficients by

$$y_{lm\,l'm}(\omega_{nlm}) \approx \frac{\sum_{k''\in K_k} c_{kk''} L_{k'k''} U_{k''}(R)}{\sum_{k''\in K_k} c_{kk''} L_{kk''} U_{k''}(R)} \in \mathbb{C}\,. \tag{13}$$

Equation (13) provides the model which relates the observations to the coupling coefficients that have to be determined in order to infer the meridional flow.

3 Method

For the inversion of the meridional flow we first estimate the amplitude ratios between coupling modes from global oscillations time series using Eq. (12). From the amplitude ratios we estimate the b-coefficients by a least-squares fitting routine using Eqs. (13), (2), and (7). The estimation of the radial flow strength of the meridional flow from the b-coefficients is a linear inversion problem given by Eq. (9) that is here solved by the SOLA inversion technique which is also used in helioseismology for the inversion of the differential rotation (Pijpers & Thompson 1994). The radial flow strength is estimated for a discrete grid of target position $\{r_j\}_{j=1,\ldots,J}$ covering the radii $0.7 \le r_j/R \le 1$, where R is the solar radius. From the radial flow strength we estimate the horizontal flow strength by means of the mass conservation Eq. (6). This is performed by a local polynomial fit of degree $N = 3$ to the radial flow component to accomplish the estimation of derivatives. The estimation error of the flow speed is obtained from the estimation errors of the amplitude ratios and the rules of error propagation through the inversion procedure.

4 Results

4.1 Application to simulated data

We have tested our inversion approach in a simulation study on artificial amplitude ratios. For this purpose we used a simple meridional flow model which consists of a single $s = 2$ flow component. It has a single flow cell on each hemisphere which ranges between the solar surface and the bottom of the convection zone. The horizontal flow strength is adapted to a surface flow speed of approximately 16 m/s for $\theta = 45°$. The model and the parameters

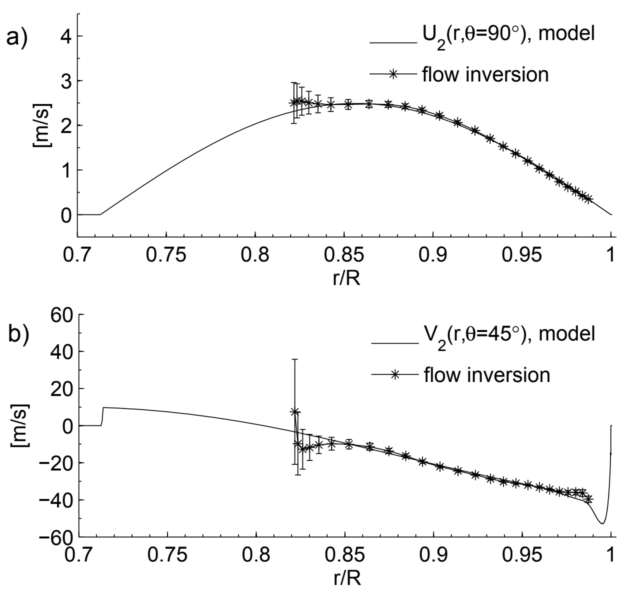

Fig. 1: Simulated and estimated *a*) radial flow speed U_2 at the equator and *b*) horizontal flow speed V_2 at mid-latitude with 1σ-error bars as a function of r/R for the $s = 2$ flow component. Positive values of $U_2(V_2)$ indicate an outward (southward) directed flow.

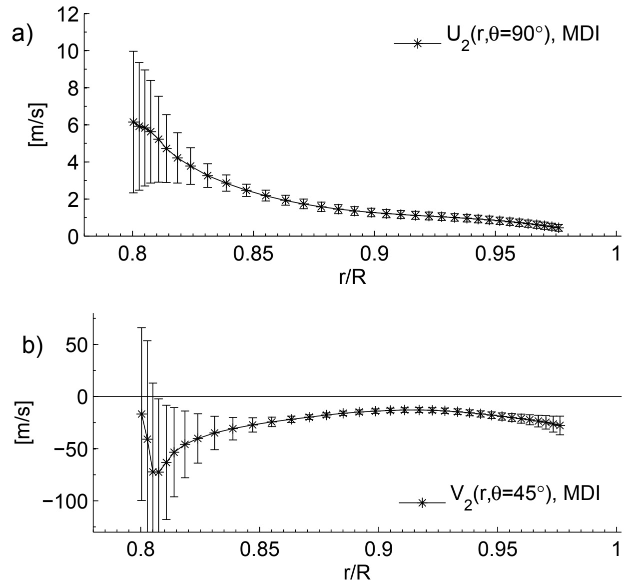

Fig. 2: *a*) Radial flow speed U_2 at the equator and *b*) horizontal flow speed V_2 at mid-latitude with 1σ-error bars as a function of r/R for the $s = 2$ component of the meridional flow estimated from data from MDI. Positive values of $U_2(V_2)$ indicate an outward (southward) directed flow.

are chosen to match approximately subsurface measurements of the meridional flow which indicate the presence of a dominant meridional flow component of low degree with a single cell per hemisphere (Hathaway 1996). The coupling coefficients and poloidal flow kernels are computed using the eigenfunctions and eigenfrequencies from solar model S (Christensen-Dalsgaard et al. 1996) for modes with $1 \leq l \leq 200$. The amplitude ratios are computed from the coupling coefficients and the leakage matrix of MDI provided by J. Schou, HMI/SDO Team, Stanford (private communication). White Gaussian noise is added to the amplitude ratios such that the relative noise level is 1 %.

The radial profiles for the simulated flow speed U_2 and V_2 and the respective inversion results for the $s = 2$ flow component are shown in Fig. 1 for selected co-latitudes θ corresponding to the equator and mid-latitude. The dependency of the $s = 2$ flow component on co-latitude θ is given by

$$U_2(r, \theta) = u_2(r) Y_2^0(\theta, \phi) = u_2(r) \frac{1}{4} \sqrt{\frac{5}{\pi}} (3 \cos^2 \theta - 1) \tag{14}$$

and

$$V_2(r, \theta) = v_2(r) \frac{\partial Y_2^0(\theta, \phi)}{\partial \theta} = -\frac{3}{2} \sqrt{\frac{5}{\pi}} v_2(r) \cos \theta \sin \theta \, . \tag{15}$$

Only well localized inversion kernels are evaluated which are obtained for target positions between $0.82 \leq r/R \leq 0.99$. The 1σ-error for the radial flow speed at the equator ranges between 0.01 m/s near the surface and 0.5 m/s at larger depths. The 1σ-error of the horizontal component increases from 1.7 m/s near the surface to 28 m/s at large depth. Overall the inversion result is in a good agreement with the model.

4.2 Meridional flow from MDI data

The method was applied to global oscillation time series from MDI covering the years 2004–2010. We analyzed the degrees $1 \leq l \leq 200$ and evaluated the inversion for the $s = 2$ flow component. The inversion results for the radial flow speed U_2 and for the horizontal flow speed V_2 for selected co-latitudes are shown in Fig. 2.

We evaluated the flow only at target positions that correspond to well localized inversion kernels from the SOLA inversion. Well localized inversion kernels are obtained on the interval $0.82 \leq r/R \leq 0.976$ that corresponds to depths between ≈16.6–125 Mm. The radial flow tends to zero near the surface and grows approximately linear with depth. The horizontal flow component is approximately constant over the accessible radii. The 1σ-error of the radial flow speed at the equator ranges between 0.19–3.8 m/s; for the horizontal component at mid-latitude the 1σ-error ranges between 2.1–83 m/s. A return flow for the investigated flow component and depths is not observed. The angular dependency of the flow speeds U_2 and V_2 at 16.6 Mm depth are shown in Fig. 3. The radial flow is directed outward at the equator and inward at the poles. The horizontal flow is directed polewards on both hemispheres. The radial flow speed at 16.6 Mm depth is very small, about 0.45 ± 0.19 m/s at the equator. The maximum horizontal flow strength at 16.6 Mm

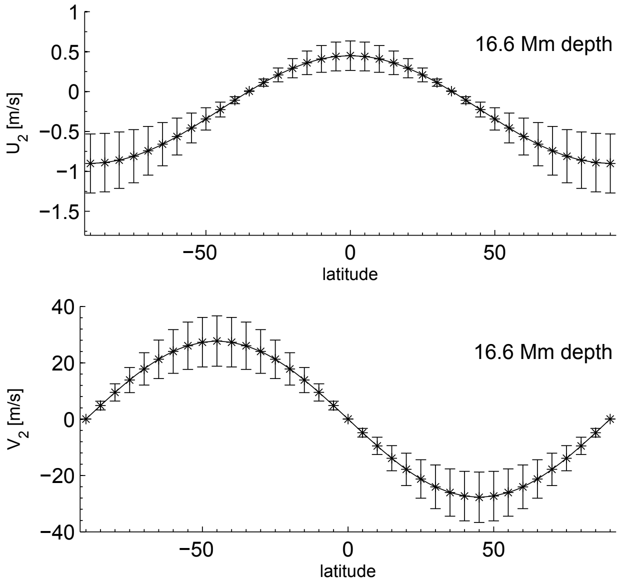

Fig. 3: *a*) Radial flow speed U_2 and *b*) horizontal flow speed V_2 with 1σ-error bars as a function of latitude at 16.6 Mm depth ($r = 0.976\,R$) from the photospheric surface estimated from data from MDI. Positive values of $U_2(V_2)$ indicate an outward (southward) directed flow.

depth is found at $\theta = 45°$ with 28 ± 9 m/s. This value is in accordance to the horizontal component of the meridional flow obtained from local helioseismic measurements at this depth (Komm et al. 2005).

5 Discussion and conclusion

We have presented a promising new approach for the global seismic investigation of the meridional flow at large depths. The approach is based on the analysis of the perturbation of the eigenfunctions that occurs in first order which is in contrast to the perturbation of mode frequency which is of second order. The new approach allows to infer the radial as well as the horizontal flow component. By means of simulations and applications to global oscillation time series from MDI we have demonstrated that we are able to infer the $s = 2$ flow component between depths of ≈16.6–125 Mm. For the Sun we find that the $s = 2$ component is directed poleward for all accessible depths. A return flow is not yet observed. Magnitude and direction of the inversion results for the $s = 2$ flow component obtained from MDI data are in accordance to subsurface measurements of the flow from ring-diagram analysis. Flow inversion at larger depths are, at least from a theoretical point of view, conceivable by the analysis of further mode couplings and data. This will be done in our future work. We further aim to evaluate further flow components of higher degree s that correspond to smaller flow cells structures.

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