Statistics and Numerics

Lecture Prof. Dr. Jens Timmer

Exercises Helge Hass, Mirjam Fehling-Kaschek

Exercise Sheet Nr. 10

Exercise 1: Brownian motion

Consider the one dimensional Brownian motion process

$$x(t) = x(t-h) + a \cdot \varepsilon$$

with $\varepsilon \sim N(0,1)$, $t \in [0,100]$, scaling constant a and integration step size h.

a) Simulate 1000 realisations of Brownian motion for $x(0) = 0$, $h = 1$ and $a = 1$. Plot the trajectories of the first five realisations and the mean value, variance and standard deviation of the realisations as a function of time.

b) Repeat the exercise for $h = 0.1$ and $h = 0.01$. Scale the noise such that the time-dependency of the variance stays unchanged.

c) Plot trajectories for a reduced sampling step size $t \in \{0,2,4,\ldots,100\}$ for the different values of the integration step size h.

d) Write down the stochastic differential equation of Brownian motion physicist's like.

Exercise 2: Stochastic Van-der-Pol oscillator

Consider the Van der Pol oscillator

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \mu(1-x_1^2)x_2 - x_1 + a\varepsilon \end{aligned}$$

with noise $\varepsilon \sim N(0,1)$.

- Simulate solutions of the oscillator using the Euler method for $t \in [0,50]$ with sampling size $\Delta t = 0.1$, $\mu \in \{1,5\}$, $a = 0$ and $h = 0.01$. Plot the result in the configuration and phase space.
- Evaluate the influence of the noise by comparing solutions for $a \in \{0, 0.5, 1, 2\}$. *Hint:* remember how to treat the noise term (see lecture).
- Test how the integration step size h affects the solutions.

helge.hass@fdm.uni-freiburg.de mirjam.fehling@physik.uni-freiburg.de
http://jeti.uni-freiburg.de/vorles_stat_num/vorles_stat_num.html